RevoLUT : Rust Efficient Versatile Oblivious Look-Up-Tables

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ABSTRACT

In this paper we present RevoLUT, a library implemented in Rust that reimagines the use of Look-Up-Tables (LUT) beyond their conventional role in function encoding, as commonly used in TFHE's programmable boostrapping. Instead, RevoLUT leverages LUTs as first class objects, enabling efficient oblivious operations such as array access, elements sorting and permutation directly within the table. This approach supports oblivious algortithm, providing a secure, privacy-preserving solution for handling sensitive data in various applications.

KEYWORDS

Privacy, Homomorphic encryption, Oblivious algorithm, Library

1 INTRODUCTION

For many years, fully homomorphic encryption was limited to vanilla arithmetic operations - multiplication and addition of ciphertexts. Non-linear functions, such as ReLu, had to be approximated using polynomial interpolation. A breakthrough came with the development of FHEW-like schemes [8, 12, 14], which introduced new bootstrapping operations that enabled an important mechanism: programmable bootstrapping (PBS). During a PBS operation, any function, linear or non-linear, can be evaluated at no cost. The outputs of the function are discretized in a Look-Up-Table (LUT) and then applied during the PBS on the bootstrapped data. Other works are also trying to apply this paradigm to other encryption schemes such as the CKKS scheme [9] like in [2] but, to the best of our knowledge, at the moment PBS is only available in pratice in TFHE.

Look-Up-Tables are a well-established concept in computer science, where they serve as data structures that store precomputed values to avoid expensive runtime computation. However, in the context of fully homomorphic encryption, LUTs have evolved further their traditional role. They have become essential building blocks for many applications ranging from storing an activation function in neural networks [6, 7] to designing a 8-bit FHE processor abstraction [17].

With RevoLUT, we take the innovation a step further by introducing LUTs as first-class objects. The library offers a range of oblivious mechanisms to manipulate, compute, access, permute, and even sort these objects. This approach provides a novel abstraction, empowering engineers to design efficient oblivious algorithms for computing on encrypted data.

2 PRELIMINARIES

2.1 Notation

Let p be a power of 2. We denote by $\mathbb{Z}_p$ the set of messages and by $[\![m]\!]$ the TFHE encryption of a message $m \in \mathbb{Z}_p$. For N a power of 2, we define $\mathcal{R}$ as the quotient ring $\mathbb{Z}[X]/(X^N+1)$ and $\mathcal{R}_q$ as the same ring modulo q, that is $\mathbb{Z}_q[X]/(X^N+1)$. Unless otherwise specified, all operations in this paper are performed in the ring $\mathcal{R}_q$. We also make use of the Kronecker delta function $\delta_{i,j}$, which equals 1 when $i = j$ and 0 otherwise. Using this notation, we can define the one-hot encoding of an integer i as the bit vector $\delta_i = (\delta_{i,0}, \ldots, \delta_{i,p-1}) \in \{0,1\}^p$, which contains a single 1 at index i and 0s elsewhere. Other notations are defined in the text when needed.

2.2 The TFHE Cryptosystem

The TFHE encryption scheme, proposed in 2016 [10, 11], is based on the security of the Learning With Errors (LWE) problem and its ring variant, the Ring-LWE (RLWE) problem.

2.2.1 Ciphertext Types. In TFHE, several types of ciphertexts are defined depending on the nature of the plaintext and the encryption method employed. A commonly used type in this paper is the General LWE (GLWE) ciphertext, defined as follows:

GLWE Ciphertexts. A message $m \in \mathbb{Z}_p$ can be encrypted under the secret key $s = (s_0, \ldots, s_{k-1}) \xleftarrow{\$} \mathbb{Z}_2^k$ as a GLWE ciphertext $(a, b) \in \mathcal{R}_q^{k+1}$, where $a = (a_0, \ldots, a_{k-1}) \xleftarrow{\$} \mathcal{R}_q^k$ and $b = \sum_{i=0}^{k-1} a_i \cdot s_i + \Delta m + e$, with $\Delta = \frac{q}{p}$ and e being a noise term sampled from a Gaussian distribution. The vector a is called *mask* and b *body*.

Specifically, when $N = 1$, the ciphertext is referred to as an LWE ciphertext. When $k = 1$ and $N > 1$, it is termed an RLWE ciphertext. In this case, an LWE ciphertext encrypts a message in $\mathbb{Z}_q$, while an RLWE ciphertext encrypts a polynomial in $\mathbb{Z}_q[X]$ modulo $X^N + 1$.

LUT Ciphertexts. Additionally, [4] introduced the concept of Look-Up-Table (LUT) ciphertexts, which are essentially RLWE ciphertexts that include some redundancy. A Look-Up-Table in TFHE is a vector $(m_i)_{0 \le i < p}$ of $\mathbb{Z}_p$ elements represented as a polynomial

$M(X) \in \mathcal{R}_q$ of the form:

$$M(X) = \sum_{i=0}^{p-1} \sum_{j=0}^{\frac{N}{p}-1} m_i X^{i\frac{N}{p}+j}$$

This polynomial is then encrypted as an RLWE ciphertext to form a LUT ciphertext as illustrated in Figure 1.

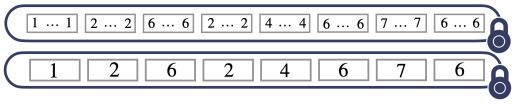

Fig. 1. Illustration of a RLWE ciphertext (top) with redundancy shown in gray boxes, which implements a LUT ciphertext (bottom) where each box represents an element in $\mathbb{Z}_p$ (here $p = 8$).

In this paper, ciphertexts are denoted within brackets to indicate their type. For instance, $[\![M]\!]_{\text{LUT}} = [\![m_0, \ldots, m_{p-1}]\!]_{\text{LUT}}$ represents the message $M = (m_0, \ldots, m_{p-1})$ encrypted as a LUT ciphertext, while $[\![m]\!]_{\text{LWE}}$ is an LWE ciphertext and $[m]_{\text{LWE}}$ is a trivially encrypted LWE ciphertext (that is a ciphertext whose mask and noise are set to 0).

2.2.2 TFHE's operations. TFHE provides several building blocks for performing homomorphic operations on ciphertexts. The main operations used in this paper are:

- **Blind Rotation (BR):** $([\![\star]\!]_{\text{LWE}}, [\![\star]\!]_{\text{LUT}}) \rightarrow [\![\star]\!]_{\text{RLWE}}$.
  This operation is used to privately rotate the polynomial $M(X)$ (encrypted as an RLWE ciphertext) by $[\![i]\!]_{\text{LWE}}$ coefficients.
- **Sample Extraction (SE):** $(\star, [\![\star]\!]_{\text{RLWE}}) \rightarrow [\![\star]\!]_{\text{LWE}}$.
  This operation extracts a coefficient from the polynomial $M(X) = \sum_{i=0}^{N-1} m_i X^i$ encrypted as an RLWE ciphertext, resulting in an LWE ciphertext $[\![m_j]\!]_{\text{LWE}}$. The LWE ciphertext is generated by selecting specific coefficients from the RLWE input.
- **Key Switching (KS):** $[\![\star]\!]_{\text{LWE}} \rightarrow [\![\star]\!]_{\text{LWE}}$.
  This operation switches the secret key or parameters of an LWE ciphertext to new ones by homomorphically re-encrypting the ciphertext with a different key.
- **Public Functional Key Switch (PFKS):** $\{[\![\star]\!]_{\text{LWE}}\} \rightarrow [\![\star]\!]_{\text{RLWE}}$.
  Introduced in [12] (Algorithm 2), this operation allows for the compact representation of multiple LWE ciphertexts into a single RLWE ciphertext, effectively packing several LWE ciphertexts into one.

The redundancy in a LUT ciphertext is mainly important to guarantee the correctness of the bootstrapping operation. Indeed, the LWE ciphertext used in the Blind Rotation operation serves as an index to select the correct coefficient from the LUT ciphertext. However, this LWE ciphertext incorporates a gaussian noise e which is bounded by N/p after the so-called Modulus Switching operation (see [13] for more details). This bound gives exactly the size of the redundancy of the coefficients in the RLWE ciphertext implementing the LUT. These sequences of consecutive coefficients in the RLWE ciphertext implementing a LUT are generally called *boxes*. During the (functional) bootstrapping operation, each box corresponds to a specific message m_i of the LUT ciphertext. When the Blind Rotation is performed, $[\![i]\!]_{\text{LWE}}$ points to the i-th box containing the message m_i in the LUT. Thus, the redundancy ensures that, despite the random error present in $[\![i]\!]_{\text{LWE}}$, the Sample Extraction operation will still correctly select the message m_i as long as the noise e is smaller than the redundancy.

2.3 Scaling

Due to the difficulty of scaling the cryptographic parameter size, (on commodity hardware, we usually have $p < 256$) it is common practice to break down larger message values into digits modulo p. In this paper we'll call the encryption of an N digit (modulo p) integer $m = \sum_{i=1}^{N} m_i p^{N-i}$ as $[\![m]\!]_{\text{LWE}_N}$, or simply $[\![m_1, \ldots, m_N]\!]_{\text{LWE}_N}$.

For LUTs, there are two variables we are interested in scaling up beyond the parameter p. The first one is the precision of the index, or the number of indexed values. We can go from indexing p values to indexing p^M for some $M \in \mathbb{N}$ using the multivariate LUT evaluation devised by [15]. This is essentially indexing a multi-dimensionnal LUT with M indices, but can equivalently be seen as indexing the LUT with a M-digit index encrypted as a LWE_M. We will denote such structure $[\![L]\!]_{\text{LUT}_M}$ or $[\![(m_{i_1,\ldots,i_M})_{0 \le i_1,\ldots,i_M < p}]\!]_{\text{LUT}_M}$.

Secondly, we may wish to increase the precision of output values of the LUT. This can be achieved by simple repetition of the structure; that is, construct multiple LUTs, one for each digit of output needed. Combining with the previous notation, we will note a structure indexed by M-digit integers holding N-digit values like $[\![L]\!]_{\text{LUT}_{M,N}}$ or $[\![(m_{i_1,\ldots,i_M})_{0 \le i_1,\ldots,i_M < p}]\!]_{\text{LUT}_{M,N}}$ where each message is an N-digit integer like $m_{i_1,\ldots,i_M} = (m^j_{i_1,\ldots,i_M})_{1 \le j \le N}$.

3 REVOLUT'S OPERATIONS

The aforementioned operations implemented in [18] led us to design a library named RevoLUT that leverages the LUT ciphertexts and enables manipulating data obliviously in those recipients. In the following, we present the main operations implemented in RevoLUT and used in our sorting algorithm. We refer the interested reader to [1] for more details and more operations.

3.1 Reading operations

Look-Up-Tables seen as ciphertexts of arrays and TFHE's operations offer some reading and writing operations that can be used to implement oblivious algorithms. The operations are shown first for simple LUTs, but then generalized to LUT_M. Going from LUT_M to $\text{LUT}_{M,N}$ is simply repeating the same procedure N times (i.e. for each output digit).

3.1.1 Blind Array Access (BAA). Introduced in [3], this operation is the most basic reading operation on a LUT ciphertext. It is basically the *programmable bootstrapping* of TFHE. Given a LUT ciphertext of an array and a LWE ciphertext of an index, BAA returns a LWE ciphertext of the array element at the given ciphered index.

Algorithm 1: Blind Array Access (BAA)

Input : An array $[\![L = (m_0, \ldots, m_{p-1})]\!]_{\text{LUT}}$
An index $[\![i]\!]_{\text{LWE}}$
Output: An encrypted value $[\![m_i]\!]_{\text{LWE}}$

1 $[\![rotated]\!]_{\text{LUT}} \leftarrow BR([\![i]\!]_{\text{LWE}}, [\![L]\!]_{\text{LUT}})$
2 $[\![m_i]\!]_{\text{LWE}} \leftarrow SE(0, [\![rotated]\!]_{\text{LUT}})$
3 **return** $[\![m_i]\!]_{\text{LWE}}$

This method allows us to blindly select an element from a given ciphertext array, and will serve as a building block for next procedures.

3.1.2 Blind Matrix Access (BMA). Introduced in [4], this operation is the generalization of BAA to matrices. Given a matrix encoded as a row first vector of LUT ciphtertexts, and a pair of indices as LWE ciphertexts, BMA returns the value stored at the row and column as a LWE ciphertext.

Algorithm 2: Blind Matrix Access (BMA)

Input : A matrix $[\![L = (m_{i,j})_{0\leq i,j<p}]\!]_{\mathrm{LUT}_2}$
A pair of indices $[\![r, c]\!]_{\mathrm{LWE}_2}$
Output: The value $[\![m_{r,c}]\!]_{\mathrm{LWE}}$

for $i \leftarrow 0$ **to** $p-1$ **do**
  $[\![L_i]\!]_{\mathrm{LUT}} \leftarrow PFKS(([\![m_{i,j}]\!]_{\mathrm{LWE}})_{0\leq j<p})$
  $[\![m_{i,c}]\!]_{\mathrm{LWE}} \leftarrow BAA([\![L_i]\!]_{\mathrm{LUT}}, [\![c]\!]_{\mathrm{LWE}})$
end
$[\![(m_{i,c})_{0\leq i<p}]\!]_{\mathrm{LUT}} \leftarrow PFKS(([\![m_{i,c}]\!]_{\mathrm{LWE}})_{0\leq i<p})$
$[\![m_{r,c}]\!]_{\mathrm{LWE}} \leftarrow BAA([\![(m_{i,c})_{0\leq i<p}]\!]_{\mathrm{LUT}}, [\![r]\!]_{\mathrm{LWE}})$
return $[\![m_{r,c}]\!]_{\mathrm{LWE}}$

In the previous pseudo-code, calls to SampleExtract have been ellided for clarity, and because their cost is minimal.

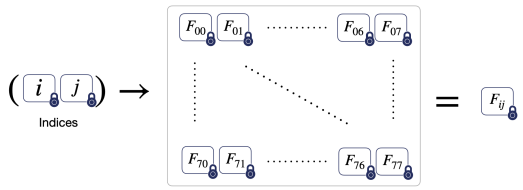

Fig. 2. An illustration of the Blind Matrix Access (BMA) operation. The matrix is encoded as a row first vector of LUT ciphertexts, and the indices are encrypted as LWE ciphertexts. The operation returns the value at the given row and column as a LWE ciphertext.

3.1.3 Blind Tensor Access (BTA). This operation is a generalization of BMA to arbitrarily higher dimension arrays. Similarly to the tree-based multivariate LUT evaluation of [15], we can fetch values from larger LUT, which is useful to represent multi-dimensional encrypted arrays, or to increase the precision of the index.

The idea is to construct a LUT_1 from the given LUT_M by fixing the first index for each $j \in \mathbb{Z}_p$ and calling the procedure recursively on each of the obtained LUT_{M-1}. Then from the p LWE ciphertexts obtained pack a simple LUT via PFKS and call the Blind Array Access from Algorithm 1 on it.

3.2 Writing operations

In this section we discuss blind writing operations; taking an encrypted array, an encrypted position and an encrypted value and giving back the initial array with the value added in the given position. They all operate by first constructing a same-dimension LUT mostly empty LUT with the given value in the given position, and add it to the given array.

Algorithm 3: Blind Tensor Access (BTA)

Input : A tensor $[\![L = (m_{i_1,\ldots,i_M})_{0\leq i_1,\ldots,i_M<p}]\!]_{\mathrm{LUT}_M}$
An index $[\![i_1, \ldots i_M]\!]_{\mathrm{LWE}_M}$
Output: The value $[\![m_{i_1,\ldots,i_M}]\!]_{\mathrm{LWE}}$

if $M = 1$ **then**
  return $BAA([\![(m_i)_{0\leq i<p}]\!]_{\mathrm{LUT}}, [\![i_1]\!]_{\mathrm{LWE}})$
end
for $j \leftarrow 0$ **to** $p-1$ **do**
  $[\![L_j]\!]_{\mathrm{LUT}_{M-1}} \leftarrow [\![(m_{j,i_2,\ldots,i_M})_{0\leq i_2,\ldots,i_M<p}]\!]_{\mathrm{LUT}_{M-1}}$
  $[\![m_j]\!]_{\mathrm{LWE}} \leftarrow BTA([\![L]\!]_{\mathrm{LUT}_{M-1}}, [\![i_2, \ldots, i_M]\!]_{\mathrm{LWE}_{M-1}})$
end
return $BAA(PFKS([\![m_0]\!]_{\mathrm{LWE}}, \ldots, [\![m_{p-1}]\!]_{\mathrm{LWE}}), [\![i_1]\!]_{\mathrm{LWE}})$

3.2.1 Blind Array Add (BAAdd). Introduced in [5], BAAdd is a primitive that enables blind writing in a LUT ciphertext by adding a value to specific position. Given a LUT ciphertext, an encrypted index i, and an encrypted value x, BAAdd adds x to the element at position i while keeping all other elements unchanged. A related operation, Blind Array Assignment, could be implemented by first reading the current value at position i using BAA, subtracting it from $[\![x]\!]_{\mathrm{LWE}}$, and then using BAAdd with the difference. While this would enable arbitrary value assignments, it requires an additional blind rotation.

Algorithm 4: Blind Array Add (BAAdd)

Input : An array $[\![L]\!]_{\mathrm{LUT}}$
An index $[\![i]\!]_{\mathrm{LWE}}$ and a value $[\![x]\!]_{\mathrm{LWE}}$
Output: The array $[\![L + x\delta_i]\!]_{\mathrm{LUT}}$

$[\![x\delta_0]\!]_{\mathrm{LUT}} \leftarrow PFKS([\![x]\!]_{\mathrm{LWE}})$
$[\![x\delta_i]\!]_{\mathrm{LUT}} \leftarrow BR(-[\![i]\!]_{\mathrm{LWE}}, [\![x\delta_0]\!]_{\mathrm{LUT}})$
return $[\![L]\!]_{\mathrm{LUT}} + [\![x\delta_i]\!]_{\mathrm{LUT}}$

A caveat of this approach is that the $[\![x\delta_i]\!]_{\mathrm{LUT}}$ is most likely misaligned due to the noise present in the rotation index. This affects the frontiers of the redundancy boxes present in LUT ciphertexts as shown in Figure 3. A way to avoid error propagation is to bootstrap the LUT by extracting every message and pack them in a fresh LUT, as described in section 3.4.

3.2.2 Blind Matrix Add (BMAdd). The Blind Array Add procedure can be generalized to 2-dimensional arrays by following the same idea of blindly constructing $x\delta_{r,c}$ a mostly empty LUT with only the desired value (x) in the desired place (r, c), and then adding it to the given LUT in place.

In the previous pseudo-code, we denote by set union the iterative construction of $[\![x\delta_{r,c}]\!]_{\mathrm{LUT}_2}$ for simplification. The idea is that one can construct a LUT_2 from p LUTs by collecting them into a row-first array, or by SampleExtracting them into a 2-dimensional array of LWE ciphertexts.

3.2.3 Blind Tensor Add (BTAdd). We can repeat this idea of lifting all LWE ciphertexts of a LUT_{M-1} via PFKS to produce a LUT_M, and blindly rotate all the LUTs to shift the whole structure by its new axis. This allows us to construct $[\![x\delta_{i_1,\ldots,i_M}]\!]_{\mathrm{LUT}}$ the mostly empty M-dimensional LUT, with a single value x at index $i_1, \ldots, i_M$.

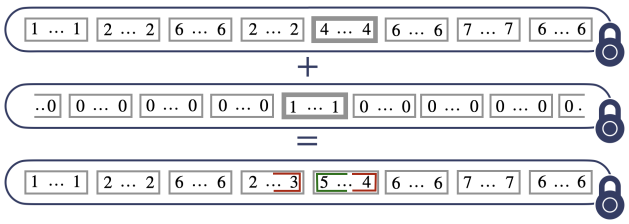

Fig. 3. Illustration of $BAAd(\llbracket 4 \rrbracket_{\text{LWE}}, \llbracket 1, 2, 6, 2, 4, 6, 7, 6 \rrbracket_{\text{LUT}}, \llbracket 1 \rrbracket_{\text{LWE}})$ with $p = 8$. The red areas at the boundaries of the redundancy boxes represent errors due to the noise in the LWE encryption of $\llbracket 4 \rrbracket_{\text{LWE}}$. If the noise in the LWE ciphertext were zero, the boxes would be perfectly aligned. However, since we have no control over this noise, except that it does not exceed $(N/2p)$, only the center of the boxes remains accurate.

Algorithm 5: Blind Matrix Add (BMAdd)

Input : A matrix $\llbracket L \rrbracket_{\text{LUT}_2}$
A pair of indices $\llbracket r, c \rrbracket_{\text{LWE}_2}$
A value $\llbracket x \rrbracket_{\text{LWE}}$
Output : The matrix $\llbracket L + x\delta_{r,c} \rrbracket_{\text{LUT}_2}$

$\llbracket x\delta_r \rrbracket_{\text{LUT}} \leftarrow BR(-\llbracket r \rrbracket_{\text{LWE}}, PFKS(\llbracket x \rrbracket_{\text{LWE}}))$
$\llbracket x\delta_{r,c} \rrbracket_{\text{LUT}_2} \leftarrow \{\}$
for $\llbracket b \rrbracket_{\text{LWE}} : \llbracket x\delta_r \rrbracket_{\text{LUT}}$ **do**
  $\llbracket R \rrbracket_{\text{LUT}} \leftarrow BR(-\llbracket c \rrbracket_{\text{LWE}}, PFKS(\llbracket b \rrbracket_{\text{LWE}}))$
  $\llbracket x\delta_{r,c} \rrbracket_{\text{LUT}_2} \leftarrow \llbracket x\delta_{r,c} \rrbracket_{\text{LUT}_2} \cup \llbracket R \rrbracket_{\text{LUT}}$
end
return $\llbracket L \rrbracket_{\text{LUT}_2} + \llbracket x\delta_{r,c} \rrbracket_{\text{LUT}_2}$

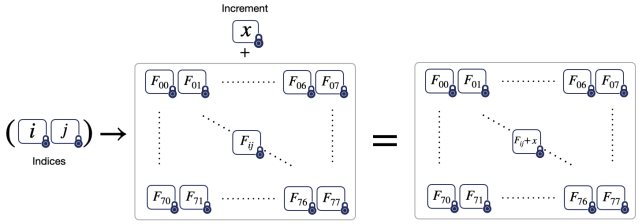

Fig. 4. Illustration of the Blind Matrix Add (BMAdd) operation. The matrix is first encoded as a row first vector of LUT ciphertexts, and the indices are encrypted as LWE ciphertexts. The operation returns the matrix with the increment value x at the given row and column.

As previously mentioned, assignement instead of addition can be implemented by preceding the addition with a prefetch and negation. However, it is worth noting too that in general, addition between LWE_N has to respect carries between digits, which can be implemented for example by blind evaluation of a public $\text{LUT}_{2,1}$ representing the binary predicate $(x, y) \mapsto (x+y \geq p)$. This process can be costly and thankfully avoided in the case of assignement as subtracting a value to itself and then adding a digit is guaranteed not to overflow.

Algorithm 6: Blind Tensor Add (BTAdd)

Input : A tensor $\llbracket L \rrbracket_{\text{LUT}_M}$
An index $\llbracket i_1, \ldots, i_M \rrbracket_{\text{LWE}_M}$
A value $\llbracket x \rrbracket_{\text{LWE}}$
Output : The tensor $\llbracket L + x\delta_{i_1,\ldots,i_M} \rrbracket_{\text{LUT}_M}$

$\llbracket x\delta_{i_1} \rrbracket_{\text{LUT}} \leftarrow BR(-\llbracket i_1 \rrbracket_{\text{LWE}}, PFKS(\llbracket x \rrbracket_{\text{LWE}}))$
for $d \leftarrow 2$ *to* M **do**
  $\llbracket x\delta_{i_1,\ldots,i_d} \rrbracket_{\text{LUT}_d} \leftarrow \{\}$
  for $\llbracket b \rrbracket_{\text{LWE}} : \llbracket x\delta_{i_1,\ldots,i_{d-1}} \rrbracket_{\text{LUT}}$ **do**
    $\llbracket R \rrbracket_{\text{LUT}} \leftarrow BR(-\llbracket i_d \rrbracket_{\text{LWE}}, PFKS(\llbracket b \rrbracket_{\text{LWE}}))$
    $\llbracket x\delta_{i_1,\ldots,i_d} \rrbracket_{\text{LUT}_d} \leftarrow \llbracket x\delta_{i_1,\ldots,i_d} \rrbracket_{\text{LUT}_d} \cup \llbracket R \rrbracket_{\text{LUT}}$
  end
end
return $\llbracket L \rrbracket_{\text{LUT}_M} + \llbracket x\delta_{i_1,\ldots,i_M} \rrbracket_{\text{LUT}_M}$

3.3 Ordering operations

In this section we discuss higher level constructions to re-arrange a given array blindly, either given an encrypted permutation vector or by sorting.

3.3.1 Blind Permutation. The permutation primitive is a fundamental building block in many privacy-preserving application (e.g cloud storage [16], private information retrieval, private set intersection, etc). Blind Permutation is an homomorphic primitive that allows to permute the elements of an encrypted vector without revealing the permutation. It leverages some building blocks of TFHE's bootstrapping operations such as Blind Rotation, Sample Extraction and Public Functional Key Switch. Specifically, the elements to be permuted are in an encrypted LUT and the permutation indices, the destination of each slot, are encrypted as an LWE ciphertext.

The permutation is done by extracting each element of the encrypted LUT as LWEs, then for each extracted element, a Public Functional Key Switch is applied to create new LUTs with the extracted element at its first position. Then we apply a Blind Rotation to the new LUTs using the LWEs representing the permutation indices. And finally, as all the other elements of the LUTS are zeros, we can sum them all to get the permuted LUT. To be sure that we can apply this indefenitely, we apply a Sample Extraction for all the elements and we create a new LWE ciphertext with the permuted elements.

Algorithm 7: Blind Permutation (BP)

Input : A LUT ciphertext $\llbracket m_0, \ldots, m_{p-1} \rrbracket_{\text{LUT}}$
A permutation vector $(\llbracket \pi_0 \rrbracket_{\text{LWE}}, \ldots, \llbracket \pi_{p-1} \rrbracket_{\text{LWE}})$
Output : A permuted LUT $\llbracket m_{\pi_0}, \ldots, m_{\pi_{p-1}} \rrbracket_{\text{LUT}}$

$\llbracket res \rrbracket_{\text{LUT}} \leftarrow \llbracket 0, \ldots, 0 \rrbracket_{\text{LUT}}$
for $i \leftarrow 0$ **to** $p - 1$ **do**
  $\llbracket m_i \rrbracket_{\text{LWE}} \leftarrow SE(i, \llbracket m_0, \ldots, m_{p-1} \rrbracket_{\text{LUT}})$
  $\llbracket r_i \rrbracket_{\text{LUT}} \leftarrow PFKS(\llbracket m_i \rrbracket_{\text{LWE}})$
  $\llbracket r_i \rrbracket_{\text{LUT}} \leftarrow BR(\llbracket \pi_i \rrbracket_{\text{LWE}}, \llbracket r_i \rrbracket_{\text{LUT}})$
  $\llbracket res \rrbracket_{\text{LUT}} \leftarrow \llbracket res \rrbracket_{\text{LUT}} + \llbracket r_i \rrbracket_{\text{LUT}}$
end
return $\llbracket res \rrbracket_{\text{LUT}}$

3.3.2 Blind Counting Sort. The idea here is to exploit the known size of the array and range of values that are given. This permits a better asymptotic complexity (linear rather than linearithmic) than possible with comparison-based sorting networks. It functions in a similar manner to the classical counting sort algorithm. First, it builds a count table, that is C_i represents how many entries in the input table have i for digit d. Then we compute the running sum of C, effectively making C a partition of positions in the input table bucketing elements by their d-th digit. Lastly it constructs the output by iterating backward over the input table, so as to preserve any previous ordering within buckets.

Algorithm 8: Blind Counting Sort (BCS)

Input : An array $[\![L = (m_{i_1,\ldots,i_M})_{0\le i_1,\ldots,i_M<p}]\!]_{\mathrm{LUT}_{M,N}}$ where $m_{i_1,\ldots,i_M} = (m^j_{i_1,\ldots,i_M})_{1\le j\le N}$ has N digits
A selection digit index $d \in [1,\ldots,N]$
Output: The given array, sorted by digit d

$[\![C]\!]_{\mathrm{LUT}_{1,M}} \leftarrow [\![0,\ldots,0]\!]_{\mathrm{LUT}_{1,M}}$
for $i = i_1,\ldots,i_M \leftarrow 0$ **to** $p^M - 1$ **do**
  // $C[m_i^d] \leftarrow C[m_i^d] + 1$
  $BAAdd([\![C]\!]_{\mathrm{LUT}_{1,M}}, [\![m_i^d]\!]_{\mathrm{LWE}}, [1]_{\mathrm{LWE}})$
end
for $i \leftarrow 1$ **to** $p - 1$ **do**
  // $C_i \leftarrow C_i + C_{i-1}$
  $[\![C_i]\!]_{\mathrm{LWE}_M} \leftarrow [\![C_i]\!]_{\mathrm{LWE}_M} + [\![C_{i-1}]\!]_{\mathrm{LWE}_M}$
end
$[\![R]\!]_{\mathrm{LUT}_{M,N}} \leftarrow [\![0,\ldots,0]\!]_{\mathrm{LUT}_{M,N}}$
for $i = i_1,\ldots,i_M \leftarrow p^M - 1$ **to** 0 **do**
  // $C[m_i^d] \leftarrow C[m_i^d] - 1$
  $BAAdd([\![C]\!]_{\mathrm{LUT}_{1,M}}, [\![m_i]\!]_{\mathrm{LWE}}, [-1]_{\mathrm{LWE}_M})$
  // $R[C[m_i^d]] \leftarrow m_i$
  $[\![C[m_i^d]]\!]_{\mathrm{LWE}_M} \leftarrow BAA([\![C]\!]_{\mathrm{LUT}_{1,M}}, [\![m_i^d]\!]_{\mathrm{LWE}})$
  $BTAdd([\![R]\!]_{\mathrm{LUT}_{M,N}}, [\![C[m_i^d]]\!]_{\mathrm{LWE}_M}, [\![m_i]\!]_{\mathrm{LWE}_N})$
end
return $[\![R]\!]_{\mathrm{LUT}_{M,N}}$

This procedure sorts single-digit values (when $d = 1$), and serves as a building block for the Blind Radix Sort for multi-digit values. Care must be taken to respect carries when implementing the blind increment and blind decrement. This can be achieved efficiently (at the cost of $M - 1$ Blind Rotation) in this case by blindly evaluating on each extra digit $[\![\delta_{p-1}]\!]_{\mathrm{LUT}}$ for increment and $[\![\delta_0]\!]_{\mathrm{LUT}}$ for decrement.

Blind Radix Sort. In order to sort multi-digit values, we can use the stability property of Counting Sort which asserts that order between elements of the same bucket is preserved. That is, if two elements were in a given order before the sort, and they share the digit under inspection, then they are guaranteed to appear in the same order in the output array. Given this, it suffices to call BCS on all digits from least to the most significant in order to sort multi-digit values.

Algorithm 9: Blind Radix Sort (BRS)

Input : A tensor $[\![L]\!]_{\mathrm{LUT}_{M,N}}$
Output: The given tensor sorted

for $d \leftarrow N$ **to** 1 **do**
  $[\![L]\!]_{\mathrm{LUT}_{M,N}} \leftarrow BCS([\![L]\!]_{\mathrm{LUT}_{M,N}}, d)$
end
return $[\![L]\!]_{\mathrm{LUT}_{M,N}}$

3.4 LUT Bootstrapping

Certain operations in RevoLUT, such as BAAdd (Algorithm 4) and BP (Algorithm 7), can disrupt the redundancy boxes as detailed in Section 3.2 and shown in Figure 3. This disruption occurs because noise in the LWE ciphertext during Blind Rotation can cause the boxes to become misaligned. To address this issue, we can extract each message from the LUT using Sample Extract and repack them into a fresh LUT before the box centers become corrupted. This repacking process ensures that the redundancy boxes in the new LUT ciphertext remain properly aligned. However it does cost a fairly expensive PFKS of p LWE ciphertexts which is often slower than a Blind Rotation.

4 EXPERIMENTAL RESULTS

In this section, we present few experimental results of our Blind operations. All experiments are performed on a computer running Ubuntu 24.04 with an Intel i9-11900KF CPU clocked at 3.5GHz and 64GB of RAM.

p	BAA	BMA	BAAdd	BP	BCS
4	7 ms	20 ms	8 ms	32 ms	116 ms
8	17 ms	56 ms	19 ms	140 ms	462 ms
16	30 ms	120 ms	34 ms	422 ms	1.34 s
32	60 ms	471 ms	70 ms	1.8 s	5.54 s
64	128 ms	2.8 s	162 ms	8.3 s	26.09 s

Table 1: Runtimes (in ms) of some blind operations of RevoLUT. The LUTs contains p elements of $\mathbb{Z}_p$

Bench Read and Write in higher dimension. The following tables show the time it takes to blindly read or write to a $\mathrm{LUT}_{M,1}$ (that is indexed by an encrypted M-digit number). To fetch a higher precision value, say a N digit value in base p, simply multiply the time taken by N. All the following benchmarks are done on single-threaded code for comparison, but the procedures are embarassingly parallel and can benefit a lot from multiple cores.

This implementation of Blind Tensor Access holds the elements as an M-dimensional array of LWE ciphertexts instead of a $(M-1)$-dimensional array of LUT ciphertexts. As such, it must execute p^M PFKS of p LWE ciphertexts more than a pre-packed variant would. As such, the times for $M = 1$ and $M = 2$ don't line up with BAA and BMA times, which are computed on pre-packed LUTs (or vectors of LUTs).

As we can see, the write operations benefits a lot from the fewer LWE ciphertexts being packed via PFKS and instead shows a time that is mostly dominated by the Blind Rotations.

	p=8		p=16		p=32	
M	size	time	size	time	size	time
1	8	32ms	16	72ms	32	416ms
2	64	256ms	256	1s	1k	13s
3	512	2s	4k	16s	32k	425s
4	4k	16s	64k	265s	1m	-

Table 2: Blind Tensor Access size and times for various p using LWE array representation

	p=8		p=16		p=32	
M	size	time	size	time	size	time
1	8	20ms	16	25ms	32	62ms
2	64	131ms	256	325ms	1k	1.6s
3	512	1s	4k	5.2s	32k	51s
4	4k	8.5s	64k	84.7s	1m	-

Table 3: Blind Tensor Add size and times for various p (without accounting for the prefetch that might happen before or the extra time that could be lost in dealing with carries)

p	M (p^M)	N (p^N)	time
16	1 (16)	1 (16)	1.34s
32	1 (32)	1 (32)	5.54s
64	1 (64)	1 (64)	26.09s
128	1 (128)	1 (128)	125s
16	2 (256)	2 (256)	209s

Table 4: Preliminary benchmarks for Blind Radix Sort, time it takes to sort p^M elements modulo p^N

For ballpark comparison, running a state of the art bitonic sort using the comparison of FheUint8 from TFHE-rs on 256 values takes roughly 1000s seconds on the same hardware.

5 APPLICATIONS OF REVOLUT

RevoLUT's read/write operations on LUT ciphertexts can be used to implement oblivious algorithms. In this section, we present some use cases where RevoLUT can be used to implement privacy-preserving algorithms, especially in the context of outsourcing computation.

5.1 PROBONITE

Proposed in [3], PROBONITE, which stands for Private One-Branch-Only Non Interactive decision Tree Evaluation, is a privacy-preserving algorithm that allows to evaluate decision trees on encrypted data in a context of Machine-Learning as a service. The proposed algorithm minimizes the number of comparisons by only evaluating the relevant branch of the decision tree. It leverages two primitives: *Blind Node Selection* which is based on Private Information Retrieval techniques and *Blind Array Access* presented in this paper. The former is used to select the appropriate branch of the decision tree to evaluate, while the latter is used to fetch the appropriate feature vector to compare to the client's request.

5.2 Private k-NN

In [5], we demonstrate how RevoLUT's Blind Counting Sort (BCS) can be leveraged as a subroutine to implement an efficient top-k algorithm in a tournament style. We demonstrate the effectiveness of this Blind top-k algorithm by implementing a privacy-preserving k-Nearest Neighbor. By using BCS instead of comparison-based sorting methods, we achieve significant performance improvements over the state-of-the-art. Traditional FHE-based sorting approaches rely on TFHE comparators that require additional bits to handle negacyclicity. In contrast, BCS operates directly with the precision of the values treated (*i.e* elements of $\mathbb{Z}_p$), as it uses counting rather than comparisons. This key difference allows our k-NN implementation to process larger batches of data more efficiently while maintaining the same accuracy. This angle of optimization is particularly interesting as it allows more room for the noise in the LWE ciphertexts and thus more leveled operations (addition, scalar multiplication etc..) with the same elements precision.

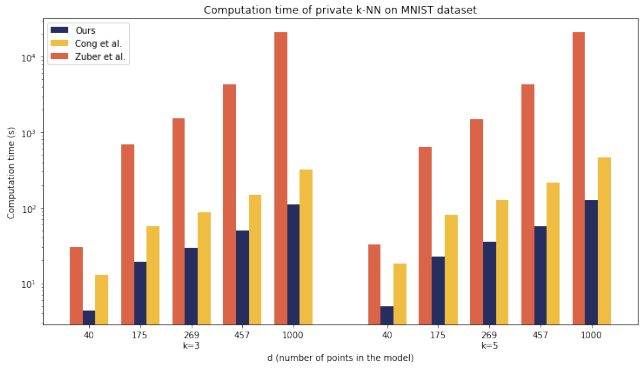

Fig. 5. Running time comparison of our private k-NN built with blind counting sort with the related works

5.3 Oblivious Turing Machine

In [4], we present a construction of an Oblivious Turing Machine using Blind Matrix Access (BMA). This construction enables clients to securely outsource both their Turing Machine and its computations to an untrusted server. The proposed construction encodes the machine's instructions as a matrix of integers in $\mathbb{Z}_p$ and the tape as a LUT ciphertext, allowing the server to execute the machine's operations without learning anything about the program or its data.

6 CONCLUSION

In this paper, we introduced RevoLUT, a Rust library built upon tfhe-rs that reimagines Look-Up-Tables as first-class objects for homomorphic encryption. Moving beyond their traditional role in function encoding during programmable bootstrapping, we demonstrated how LUTs can serve as versatile data structures enabling efficient oblivious operations. The library provides a comprehensive set of primitives for reading (BAA, BMA), writing (BAAdd), and ordering (BP, BCS) operations on encrypted data, forming a powerful toolkit for implementing privacy-preserving algorithms. Our experimental results demonstrate the practical viability of RevoLUT's operations, with reasonable execution times even for larger

LUT sizes. Through concrete applications in the context of Machine Learning and Turing Machine evaluation, we showcased how RevoLUT can be leveraged to build complex privacy-preserving systems.

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